

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

## MATHEMATICS (SYLLABUS D)

4024/02
Paper 2
May/June 2007
2 hours 30 minutes
Additional Materials: Answer Booklet/Paper
Graph paper ( 1 sheet)
Electronic calculator Geometrical instruments

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

## Section A

Answer all questions.

## Section B

Answer any four questions.
Show all your working on the same page as the rest of the answer.
Omission of essential working will result in loss of marks.
You are expected to use an electronic calculator to evaluate explicit numerical expressions. You may use mathematical tables as well if necessary.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100 .

## Section A [52 marks]

Answer all questions in this section.

1 (a) The table shows the fares charged by a taxi company.
$\$ 1.20$ per kilometre for the first 10 km then
80 cents for each additional kilometre after the first 10 km
(i) Calculate the fare for a journey of
(a) 8 km ,
(b) 24 km .
(ii) Find the length of the journey for which the fare was $\$ 16$.
(b) The table gives the times of high tides at a harbour.

| Date | May 5 | May 6 | May 7 |
| :---: | :---: | :---: | :---: |
| Times | 1000 | 1120 | 0036 |
|  | 2256 |  | 1250 |

(i) Calculate, in hours and minutes, the length of time between the high tide on May 6 and the morning high tide on May 7.
(ii) Given that low tides occurred midway between high tides, calculate the time of the low tide on the afternoon of May 5.
(c) The height of a mountain is 1800 metres.

It is suggested that this mountain has been worn away at an average rate of 0.15 mm per year.
Assuming that the suggestion is correct, calculate the height of the mountain 20 million years ago. [2]

2 The diagram represents a framework.
$B C=1.3 \mathrm{~m}, B D=1.9 \mathrm{~m}$ and $B E=1.5 \mathrm{~m}$.
$B \hat{C} D=76^{\circ}, B \hat{A} E=68^{\circ}$ and $B \hat{E} D=90^{\circ}$.


Calculate
(a) $D \hat{B} E$,
(b) $A E$,
(c) $B \hat{D} C$.

3 (a) Express as a single fraction in its simplest form $\frac{7}{6 a}-\frac{5}{9 a}$.
(b) Simplify $3 b(b-1)-2(b-2)(b+2)$.
(c) The $n$th term of a sequence, $S$, is $n^{3}+2$. The first four terms are $3,10,29$ and 66.
(i) Find the fifth term of $S$.
(ii) The first four terms of another sequence, $T$, are 4, 12, 32 and 70 . By comparing $S$ and $T$, write down
(a) the fifth term of $T$,
(b) an expression, in terms of $n$, for the $n$th term of $T$.
(d) On Monday, two girls, Jane and Susan, collected some seashells.

Jane collected $x$ shells and Susan collected 22 more than Jane.
On Tuesday, Susan gave 60 of her shells to Jane.
The table shows the numbers of shells each girl had on the two days.

|  | Jane | Susan |
| :---: | :---: | :---: |
| Monday | $x$ | $x+22$ |
| Tuesday | $x+60$ | $y$ |

(i) Write down an expression for $y$ in terms of $x$.
(ii) Given that, on Tuesday, Jane had three times as many shells as Susan,
(a) write down and solve an equation in $x$,
(b) find the total number of shells the girls collected.

4 (a) In a survey, some students were asked which of three pictures, labelled $X, Y$ and $Z$, they preferred.

The results are represented in the pie chart.

(i) Calculate the percentage of students who preferred $X$.
(ii) Find, in its simplest form, the ratio of the number of students who preferred $X$ to those who preferred $Y$.

Give your answer in the form $m: n$, where $m$ and $n$ are integers.
(iii) Given that 44 students preferred $Y$, calculate the number of students who took part in the survey.
(b) In the diagram, $A, B, C$ and $D$ lie on a circle centre $O$.

The tangent to the circle at $C$ meets the diameter $A D$ produced, at $T$.
$D \hat{O} C=66^{\circ}$.


Calculate
(i) $D \hat{A} C$,
(ii) $D \hat{T} C$,
(iii) $A \hat{D} C$,
(iv) $A \hat{B} C$.

5 Emma noted the number of letters in each of the 25 words in an examination question. The results are given in the table below.

| Number of letters | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 2 | 6 | 5 | 5 | 4 | 0 | 3 |

(a) For this distribution,
(i) write down the mode,
(ii) find the median,
(iii) calculate the mean.
(b) Emma chose one word, at random, from the 25 words.

Find the probability that this word had
(i) 5 or 6 letters,
(ii) fewer than 9 letters.
(c) Peter chose one word, at random, from the 25 words.

He then chose a second word, at random, from the remaining words.
Expressing each answer as a fraction in its lowest terms, find the probability that
(i) both words had 6 letters,
(ii) one word had 2 letters and the other had 4 letters.


In the diagram, $A$ is the point $(6,0)$ and $B$ is the point $(0,-4)$. $O A C B$ is a rectangle and $M$ is the midpoint of $B C$. $\overrightarrow{C A}=\overrightarrow{A D}$.
(a) Describe, fully, the symmetry of quadrilateral $O D A B$.
(b) Express as column vectors
(i) $\overrightarrow{C D}$,
(ii) $\overrightarrow{O C}$,
(iii) $\overrightarrow{D O}$.
(c) What type of triangle is $O C D$ ?
(d) The transformation P maps the rectangle $O B C A$ onto the quadrilateral $O B A D$. It also maps $M$ onto $M^{\prime}$.
(i) Write down the coordinates of $M^{\prime}$.
(ii) Identify the transformation P .

## Section B [48 marks]

Answer four questions in this section.
Each question in this section carries 12 marks.

7 (a) [The volume of a sphere is $\frac{4}{3} \pi r^{3}$.]
[The surface area of a sphere is $4 \pi r^{2}$.]


A wooden cuboid has length 20 cm , width 7 cm and height 4 cm .
Three hemispheres, each of radius 2.5 cm , are hollowed out of the top of the cuboid, to leave the block as shown in the diagram.
(i) Calculate the volume of wood in the block.
(ii) The four vertical sides are painted blue.

Calculate the total area that is painted blue.
(iii) The inside of each hemispherical hollow is painted white.

The flat part of the top of the block is painted red.
Calculate the total area that is painted
(a) white,
(b) red.
(b) The volume of water in a container is directly proportional to the cube of its depth. When the depth is 12 cm , the volume is $576 \mathrm{~cm}^{3}$.
Calculate
(i) the volume when the depth is 6 cm ,
(ii) the depth when the volume is $1300 \mathrm{~cm}^{3}$.

## 8 Answer the whole of this question on a sheet of graph paper.



Adam stood on a slope, 15 m from the bottom.
He rolled a heavy ball directly up the slope.
After $t$ seconds the ball was $y$ metres from the bottom of the slope.
The table below gives some values of $t$ and the corresponding values of $y$.

| $t$ | 0 | 1 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 | 5.5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 15 | 22 | 25 | 25 | 24 | 22 | 19 | 15 | 10 | 4 |

(a) Using a scale of 2 cm to represent 1 unit, draw a horizontal $t$-axis for $0 \leqslant t \leqslant 6$.

Using a scale of 2 cm to represent 5 units, draw a vertical $y$-axis for $0 \leqslant y \leqslant 30$.
On your axes, plot the points given in the table and join them with a smooth curve.
(b) Extend the curve to find the value of $t$ when the ball reached the bottom of the slope.
(c) (i) By drawing a tangent, find the gradient of the curve when $t=3.5$.
(ii) State briefly what this gradient represents.
(d) Immediately after he rolled the ball, Adam ran down the slope at a constant speed of $1.5 \mathrm{~m} / \mathrm{s}$.
(i) Write down the distance of Adam from the bottom of the slope when
(a) $t=0$,
(b) $t=4$.
(ii) On the same axes, draw the graph that represents the distance of Adam from the bottom of the slope for $0 \leqslant t \leqslant 6$.
(iii) Hence find the distance of Adam from the bottom of the slope when the ball passed him.

Diagram I


In Diagram I, the point $D$ lies on $A C$ and $N$ is the foot of the perpendicular from $C$ to $B D$. $A B=61 \mathrm{~m}, A D=30 \mathrm{~m}$ and $D C=45 \mathrm{~m}$.
Angle $B A C=41^{\circ}$.
(a) Calculate $B D$.
(b) Show that, correct to the nearest square metre, the area of triangle $B D A$ is $600 \mathrm{~m}^{2}$.
(c) Explain why $\frac{\text { area of } \triangle B C D}{\text { area of } \triangle B D A}=\frac{3}{2}$.
(d) Calculate the area of triangle $B C D$.
(e) Hence calculate $C N$.
(f)

## Diagram II



The same points $B, C, D$ and $N$ lie on a sloping plane.
The point $E$ is 15 m vertically below $C$.
The points $B, E, D$ and $N$ lie on a horizontal plane.
Diagram II represents this information.
Calculate the angle of elevation of $C$ from $N$.

10 It is given that $y=\frac{3 x^{2}-12}{5}$.
(a) Find $y$ when $x=-3$.
(b) Find the values of $x$ when $y=0$.
(c) For values of $x$ in the range $-3 \leqslant x \leqslant 2$, write down
(i) the largest value of $y$,
(ii) the smallest value of $y$.
(d) Express $x$ in terms of $y$.
(e) It is also given that $y=\frac{t-3}{2}$ when $x=t$.
(i) Show that $t$ satisfies the equation $6 t^{2}-5 t-9=0$.
(ii) Solve the equation $6 t^{2}-5 t-9=0$, giving each answer correct to two significant figures.

11 (a) The diagrams show triangles $A, B, C$ and $D$.
(i) The single transformation P maps $\Delta A$ onto $\Delta B$.

Describe, fully, the transformation P .

(ii) The single transformation Q maps $\Delta A$ onto $\Delta C$.

Describe, fully, the transformation Q .

(iii) The reflection R maps $\Delta A$ onto $\Delta D$.

Find the matrix that represents the reflection R .

(b) The diagram shows the points $E(1,3), F(2,3)$ and $G(-1,3)$. An enlargement, centre $E$, maps $F$ onto $G$.

Write down

(i) the scale factor,
(ii) the coordinates of the image of $(0,4)$.
(c) $\quad \mathbf{M}=\left(\begin{array}{ll}-1 & 3 \\ -2 & 4\end{array}\right)$
(i) Find the determinant of $\mathbf{M}$.
(ii) Write down the inverse of $\mathbf{M}$.
(iii) Find the matrix $\mathbf{X}$, where $\mathbf{M X}=\binom{4}{-2}$.

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